QP CODE: 25050452

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Reg No : .....

Name : .....

MSc DEGREE (CSS)EXAMINATION, DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010102 - LINEAR ALGEBRA

2019 Admission Onwards

ECE56C7C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. If C is the field of complex numbers, which vectors in C^3 are linear combinations of (1,0,-1), (0,1,1) and (1,1,1)?
2. Let S be a linearly independent subset of a vector space V. Suppose β is a vector in V which is not in the subspace spanned by S. Then show that the set $S \cup \{\beta\}$ is linearly independent.
3. Check whether the operator T on $F^{m \times n}$ defined by $T(A) = PAQ$ is linear, where $P \in F^{m \times n}$ and $Q \in F^{n \times n}$ are fixed matrices over the field F.
4. Prove that the null space of any nonzero linear functional on a finite dimensional vector space V is a hyper space of V.
5. Let A be an $n \times n$ matrix with real entries. If $A = 0$ prove that $trace(A^T A) = 0$.
6. Show that a linear combination of n-linear functions is n-linear.
7. If A is an invertible n x n matrix over a field, show that det A ≠ 0.
8. *Let K be the ring of rational numbers and let* $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$. *Is A invertible as a matrix over K? If so, find the inverse.*
9. Find the characteristic values, if any, of the linear operator T on R^2 which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$
10. Define minimal polynomial for a linear operator on a finite dimensional vector space V . What is the minimal polynomial for the zero operator?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Define subspace spanned by a set of vectors in a vector space V. Show that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S.

12. *Find the coordinate matrix of the vector (1, 0, 1) in the basis of $\mathbb{C}^3$ consisting of the vectors (2i, 1, 0), (2, −1, 1), (0, 1 + i, 1 − i), in that order.*

13. Prove that every n-dimensional vector space V over the field F is isomorphic to the space F^n.

14. If V is a finite dimensional vector space and T is a linear operator on V prove that T is invertible if and only if $[T]_B$ is invertible, where B is any ordered basis for V.

15. If S is any subset of a finite dimensional vector space V and W is the subspace of V spanned by S then prove that $S^{00} = W$

16. If A and B are two n x n matrices over K, then show that det (AB) = (det A) (det B).

17. *Let T be a linear operator on R^3 which is represented in the standard ordered basis by the matrix*

$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. *Prove that T is diagonalizable by exhibiting a basis for R^3, each vector of which is a characteristic vector of T*

18. *If $E_1, E_2, \cdots, E_k$ are k linear operators on V which satisfy the conditions*

i. Each E_i is a projection

ii. $E_i E_j = 0$ if $i \neq j$

iii. $I = E_1 + \cdots + E_k$

and let W_i be the range of E_i then prove that $V = W_1 \oplus \cdots \oplus W_k$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Consider the 5 x 5 matrix $A = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 4 & 1 & 10 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

(a) Find an invertible matrix P such that PA is a row-reduced echelon matrix R.

(b) Find a basis for the row space W of A.

(c) Which vectors (b_1, b_2, b_3, b_4, b_5) are in W?

(d) Find the coordinate matrix of each vector (b_1, b_2, b_3, b_4, b_5) in W in the ordered basis chosen in (b).

(e) Write each vector (b_1, b_2, b_3, b_4, b_5) in W as a linear combination of the rows of A.

(f) Give an explicit description of the vector space V of all 5 x 1 column matrices X such that AX = 0.

(g) Find a basis for V.

20. Let V and W be finite dimensional vector spaces over the field F such that dim V= dim W. If $T: V \to W$ be a linear transformation, prove that the following statements are equivalent:

    (i) T is invertible.

    (ii) T is non-singular.

    (iii) T is onto.

    (iv) If $\{\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n\}$ is a basis for V then $\{T\alpha_1, T\alpha_2, T\alpha_3, \ldots\ldots, T\alpha_n\}$ is a basis for W.

    (v) There is some basis $\{\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n\}$ for V such that $\{T\alpha_1, T\alpha_2, T\alpha_3, \ldots\ldots, T\alpha_n\}$ is a basis for W

21. (a) Let n > 1 and let D be an alternating (n-1)-linear function on (n -1) x (n -1) matrices over K. For each j, $1 \leq j \leq n$, show that the function E_j defined by $Ej(A) = \sum_{i=1}^{n} (-1)^{i+j} A_{ij} D_{ij}(A)$ is an alternating n-linear function on n x n matrices A. If D is a determinant function, then show that so is each Ej.

(b) If K= $\mathbb{R}[x]$ and A= $\begin{bmatrix} x-1 & x^2 & x^3 \\ 0 & x-2 & 1 \\ 0 & 0 & x-3 \end{bmatrix}$, find $E_j(A)$ for each j.

22. 1. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V . Prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)(x - c_2)\cdots(x - c_k)$ where $c_1, c_2, \cdots, c_k$ are distinct elements of F.

    2. *Every matrix A such that $A^2 = A$ is similar to a diagonal matrix*

(2×5=10 weightage)